Uninvadable strategies for biotrophic pathogens, from dynamic games to adaptive dynamics

Frédéric Grognard, Ivan Yegorov, Julien Guegan, Ludovic Mailleret, Fabien Halkett, Pierre Bernhard

To cite this version:

Frédéric Grognard, Ivan Yegorov, Julien Guegan, Ludovic Mailleret, Fabien Halkett, et al.. Uninvadable strategies for biotrophic pathogens, from dynamic games to adaptive dynamics. 11th European Conference on Mathematical and Theoretical Biology (ECMTB), Jul 2018, Lisbon, Portugal. hal-01940820

HAL Id: hal-01940820
https://inria.hal.science/hal-01940820

Submitted on 30 Nov 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Day: T.B.A. Hour: T.B.A. Room: T.B.A.

CONTRIBUTED TALK

UNINVADABLE STRATEGIES FOR BIOTROPHIC PATHOGENS, FROM DYNAMIC GAMES TO ADAPTIVE DYNAMICS

Frédéric Grognard

frederic.grognard@inria.fr

Université Côte d'Azur, Inria, INRA, CNRS, UPMC Univ Paris 06, 2004 route des Lucioles, Sophia Antipolis, France

Joint work with Ivan Yegorov (U. of Melbourne, Australia), Julien Guégan (Inria, France), Ludovic Mailleret (INRA, France), Fabien Halkett (INRA, France) and Pierre Bernhard (Inria, France).

Keyword: Uninvadable strategy, Differential game, Adaptive dynamics, Resource allocation, Biotrophic pathogens.

ABSTRACT

In this work, we study the competition of two biotrophic fungal cohorts within a common host plant. Their identical dynamics are described by nonlinear ordinary differential equations representing the evolution of the average mycellium size M_i of each, and driven by their potentially differing resource allocation strategy over time $u_i(t)$. Indeed, biotrophic pathogens uptake resource from living host plant tissues and opt for its allocation to mycellium growth, spore (S_i) production, or a mix of both:

$$\begin{aligned}\frac{dM_i(t)}{dt} &= (1-u_i(t))\, f_i(M_1(t), M_2(t)) - g(M_i(t))\\ \frac{dS_i(t)}{dt} &= u_i(t)\, f_i(M_1(t), M_2(t))\end{aligned}$$

Each cohort's fitness is then defined as the per capita expected spore production over a season of duration T, $J_i(u_1(.), u_2(.))$. In an uninvadability context, both cohorts then stand in a zero-sum game where they try to maximize/minimize $J_1(.) - J_2(.)$ [3], in an approach complementary to [4]. The related Cauchy problem for the Hamilton-Jacobi-Isaacs equation is investigated through analytical and numerical

approaches to obtain the solution of the zero-sum state-feedback game defined above. A first property that we obtained is that, if $M_1(0) = M_2(0)$, the Nash equilibrium corresponds to identical strategies for both cohorts. These strategies most classically consist in increasing the mycellium size upto some fixed level (with $u_i(t) = 0$), then keeping the mycellium constant at that level while producing spores (with $0 < u_i(t) < 1$), and finally only producing spores (with $u_i(t) = 1$) until the end of the season.

The obtained solution was then compared to two complementary approaches: the one obtained from single-cohort optimal resource allocation [1] and the one stemming from adaptive dynamics. We noted that the solution of the optimal approach gives a similar type of solution, except that the mycellium level at which the solution settles in the intermediate phase is lower.

We then noted from the generic form of the solutions obtained in both the optimal and game solutions that two scalar traits can define these allocation strategies: the level at which the mycellium settles during the intermediate phase, and the time before the end of the season at which the sporulation-only phase starts. Computing the equilibrium, canonical equation and stochastic adaptive dynamics for the latter problem [2], we obtained results in accordance with what the solution of the game yielded, which validates the approach.

Acknowledgements: This work was supported by a grant over-seen by the French National Research Agency (ANR) as part of the "Blanc2013" program (ANR-13-BSV7- 0011, FunFit project)

References

[1] Yegorov, Y., Grognard, F., Mailleret, L., Halkett , F. (2017). *Optimal resource allocation for biotrophic phatogens.* IFAC World Congress, Toulouse.

[2] Leimar, O. (2009). *Multidimensional convergence stability.* Evolutionnary Ecology Research 11, 191-208.

[3] Bernhard, P., Grognard, F., Mailleret, L., Akhmetzhanov, A.R. (2010). *ESS for life history of cooperating consumers facing cheating mutants* 14th International Symposium on Dynamic Games and Applications, Banff, Alberta, Canada.

[4] Dieckmann, U., Heino, M., and Parvinen, K. (2006). *The adaptive dynamics of function-valued traits.* Journal of Theoretical Biology 241. 370-389.